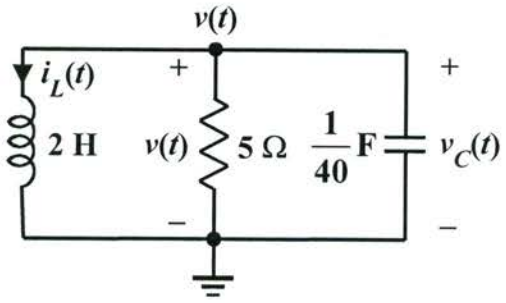
EE 2240

Problem #01

For the underdamped circuit shown, determine the voltage $v(t)$ if the initial conditions on the storage elements are $i_L(0) = 1$ A and $v_C(0) = 10$ V.



From KCL:

$$i_L(0) + \frac{1}{2}\int_0^t v\,dt + \frac{v}{5} + \frac{1}{40}\frac{dv}{dt} = 0 \qquad [1]$$

Differentiating w.r.t. t:

$$\frac{1}{2}v + \frac{1}{5}\dot{v} + \frac{1}{40}\ddot{v} = 0 \quad \text{or} \quad \ddot{v} + 8\dot{v} + 20v = 0$$

The characteristic equation is:

$$r^2 + 8r + 20 = 0 \quad \text{or} \quad (r+4)^2 + 2^2 = 0 \Rightarrow r = -4 \pm j2$$

and, therefore,

$$v(t) = e^{-4t}(K_1 \cos 2t + K_2 \sin 2t)$$

From [1] above

$$i_L(0) + \frac{1}{2}\int_0^{0^+} v(t)\,dt + \frac{v(0)}{5} + \frac{1}{40}\dot{v}(0) = 0$$

$$\Rightarrow \quad \dot{v}(0) = -40\,i_L(0) - 8\,v(0) = -120$$

Solving for K_1 and K_2, then

$$v(t) = e^{-4t}(10\cos 2t - 40\sin 2t)$$

$$= 10\,e^{-4t}(\cos 2t - 4\sin 2t) \text{ V}, \quad t \geq 0$$